Does The $\triangle I = 1/2$ Rule Hold In D and B $\to \pi\pi$ Decays?

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Abstract

Although two pion decays of K, D and B have similar isospin structures, there are dramatic differences in the ratios R_K and $R_{D,B}$ of amplitudes from $\Delta I = 3/2$ and $\Delta I = 1/2$ interactions. In $K \to \pi\pi$ decays there is the famous $\Delta I = 1/2$ rule with $R_K \approx 1/22$, whereas in $B(D) \to \pi\pi$ decays the ratios $R_{D,B}$ are of order one and therefore there is no such a rule. In this work we study decay amplitudes in $B(D) \to \pi\pi$ using QCD factorization calculations paying particular attention to discrepancies between data and theoretical estimates. Since isospin does not play a special role in factorization calculations, no $\Delta I = 1/2$ rule is expected. We find that theoretical results on the size of the amplitudes are in qualitative agreement with data. However the phases for the amplitudes are very different. We show that the effects of re-scattering between the two pions in the final state can play a crucial rule in understanding the differences between $B(D) \to \pi\pi$ and $K \to \pi\pi$ decays. We also comment on the role of isospin analysis which applies to the study of CP violation in $B \to \pi\pi$ decays.

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It is well known for many years that there is a $\Delta I = 1/2$ rule for $K \to \pi\pi$ decays. In these decays the ratio R_K of the isospin $I = 2$ amplitude for the two final pions from $\Delta I = 3/2$ interaction to the $I = 0$ amplitude from $\Delta I = 1/2$ interaction is about 1/22 which is much smaller than naive expectations. Why there is such a big difference between the two amplitudes presents a serious challenge to theory. Many attempts have been made to explain the $\Delta I = 1/2$ rule. Since the pioneer papers by Gaillard, Lee and Altarelli, Maiani[1], it was hoped that the $\Delta I = 1/2$ rule may be explained by the running of the relevant Wilson coefficients. However, the different evolution of the Wilson coefficients of the operators which correspond to $\Delta I = 1/2$ and $\Delta I = 3/2$ respectively can only result in a factor of $2 \sim 4$, as the data demands a deviation of more than 20, another factor of $6 \sim 10$ is still missing. Other theoretical scenarios have been proposed to explain the difference. For example, the final state interaction [2], the contribution of the penguin-induced operator O_6 [3] and even the diquark intermediate states [4].

Since the mesons K, D and B all have the same isospin structure, it is natural to expect that in two pion decays of B and D there are similar $\Delta I = 1/2$ rules just like in $K \to \pi\pi$ decays. It is interesting to study in more details the isospin amplitudes of $B(D) \to \pi\pi$ decays. Moreover, one hopes that the study of whether the $\Delta I = 1/2$ rule holds or not in B and D cases can provide crucial information to understand the mechanism which enforces the $\Delta I = 1/2$ rule in the K case.

The decay amplitudes for K, D and B into two pions can be parameterized in the same way as,

$$\begin{aligned}
A^{-0}(P^- \to \pi^-\pi^0) &= \sqrt{\frac{3}{2}} A_2 e^{i\delta_2}; \\
A^{+-}(P^0 \to \pi^+\pi^-) &= \frac{1}{\sqrt{3}} A_2 e^{i\delta_2} + \sqrt{\frac{2}{3}} A_0 e^{i\delta_0}, \\
A^{00}(P^0 \to \pi^0\pi^0) &= \sqrt{\frac{2}{3}} A_2 e^{i\delta_2} - \frac{1}{\sqrt{3}} A_0 e^{i\delta_0},
\end{aligned} \tag{1}$$

where P is one of the K^-, $\bar{K}^0$, D^-, D^0 and B^-, $\bar{B}^0$ mesons. A_I and δ_I are the amplitudes with the two pions in isospin "I" states and their corresponding phases, respectively.

The amplitude A_0 is induced by an interaction with isospin 1/2, whereas A_2 is induced by $I = 3/2$ interaction. The $\Delta I = 1/2$ rule refers to the fact that the ratio A_2/A_0 is much smaller than one. Using the above information one can write this ratio in terms of branching

ratios,

$$R_P = |\frac{A_2^{(P)}}{A_0^{(P)}}| = [\frac{\Gamma_P^{+0}}{\frac{3}{2}(\Gamma_P^{+-} + \Gamma_P^{00}) - \Gamma_P^{+0}}]^{1/2}. \tag{2}$$

TABLE I: Branching ratios. $D \to \pi\pi$ and $B \to \pi\pi$ are from Ref.[5] and Ref.[6], respectively.

	Data	naive factorization	QCD factorization
$BR(D^+ \to \pi^+\pi^0)$	$(2.6 \pm 0.7) \times 10^{-3}$	2.97×10^{-3}	2.88×10^{-3}
$BR(D^0 \to \pi^+\pi^-)$	$(1.38 \pm 0.05) \times 10^{-3}$	2.02×10^{-3}	2.08×10^{-3}
$BR(D^0 \to \pi^0\pi^0)$	$(8.4 \pm 2.2) \times 10^{-4}$	5.51×10^{-6}	3.65×10^{-6}
$BR(D^0 \to \pi^+\pi^- + \pi^0\pi^0)$	$(2.22 \pm 0.23) \times 10^{-3}$	2.02×10^{-3}	2.08×10^{-3}
R_D	0.65 ± 0.13	0.79	0.75
$BR(B^+ \to \pi^+\pi^0)$	$(5.5 \pm 0.6) \times 10^{-6}$	6.59×10^{-6}	5.84×10^{-6}
$BR(B^0 \to \pi^+\pi^-)$	$(4.6 \pm 0.4) \times 10^{-6}$	8.23×10^{-6}	8.38×10^{-6}
$BR(B^0 \to \pi^0\pi^0)$	$(1.51 \pm 0.6) \times 10^{-6}$	2.29×10^{-7}	2.48×10^{-7}
$BR(B^0 \to \pi^+\pi^- + \pi^0\pi^0)$	$(6.11 \pm 0.52) \times 10^{-6}$	8.46×10^{-6}	8.63×10^{-6}
R_B	1.11 ± 0.20	0.96	0.84

The branching ratios for $D \to \pi\pi$ have been measured to good precision[5]. Recently the $B \to \pi\pi$ branching ratios have also been measured[6]. In Table I we list the branching ratios of the $\pi\pi$ decays of D and B. With these measured branching ratios, one can easily check if there are $\Delta I = 1/2$ rule in B and D to $\pi\pi$ decays. We obtain

$$R_D = 0.67 \pm 0.13; \quad R_B = 1.11 \pm 0.20. \tag{3}$$

It is obvious that the $\Delta I = 1/2$ rule is violated in $B(D) \to \pi\pi$ decays. The situation is very different from that in $K \to \pi\pi$ decays. It is therefore important if one can understand the situation by studying these decays more carefully.

To have a full understanding of the situation, one also needs to consider phase shifts. From eq. (1) one obtains the isospin phase differences $\cos\delta_P = \cos(\delta_0 - \delta_2)_P$ as

$$\cos\delta_P = \frac{3|A^{+-}|^2 - 6|A^{00}|^2 + 2|A^{-0}|^2}{4\sqrt{3}|A^{-0}|\sqrt{|A^{+-}|^2 + |A^{00}|^2 - 2|A^{-0}|^2/3}}. \tag{4}$$

Using experimental data, we obtain

$$\cos\delta_D = 0.13 \pm 0.06, \quad \text{for} \;\; D \to \pi\pi,$$
$$\cos\delta_B = 0.58 \pm 0.20, \quad \text{for} \;\; B \to \pi\pi. \tag{5}$$

These results indicate that the difference of the phase shifts δ_P is sizeable. In the above we have neglected CP violation in the decay amplitude which is applicable for $D \to \pi\pi$ decays. If CP violation is sizeable which may happen in $B \to \pi\pi$ decays, one needs to replace $\cos\delta_P$ by $\cos(\delta_P + \phi_P)$ with ϕ_P being the CP violating phase difference in the decay amplitudes. We will come back to this later.

We first discuss the quantity R_P. In the past few years progresses have been made in the calculations of a heavy meson decays into two light mesons based on QCD factorization. Several ways of calculating $B \to \pi\pi$ have been developed with different methods treating non-purterbative quantities involved[7, 8, 9, 10]. There are also many model-independent studies about the amplitudes for $B \to \pi\pi$ decays based on isospin and/or SU(3) symmetries[13, 14]. We will take QCD improved factorization as the theory to compare with data. We obtain the isospin amplitudes in the following

$$\begin{aligned} A_0 &= -\{\sqrt{\frac{2}{3}}\lambda_u'(a_1 - \frac{a_2}{2}) + \sqrt{\frac{3}{2}}\lambda_p'[a_4^p + \frac{a_7}{2} - \frac{a_9}{2} + \frac{a_{10}^p}{2} + \gamma_X^\pi(a_6^p + \frac{a_8^p}{2})]\}A_{\pi\pi} \\ &\quad -\sqrt{\frac{3}{2}}[\lambda_u' b_1 + (\lambda_u' + \lambda_c')(b_3 + 2b_4 - \frac{b_3^{EW}}{2} + \frac{b_4^{EW}}{2})]B_{\pi\pi}, \\ A_2 &= -\frac{1}{\sqrt{3}}[\lambda_u'(a_1 + a_2) + \frac{3}{2}\lambda_p'(-a_7 + \gamma_X^\pi a_8^p + a_9 + a_{10}^p)]A_{\pi\pi}. \end{aligned} \tag{6}$$

In the above expressions, $\lambda_q' = V_{qb}V_{qd}^*$, $A_{\pi\pi} = i(G_F/\sqrt{2})(m_B^2 - m_\pi^2)F_+^{B\to\pi}(0)f_\pi$, and $\gamma_X^\pi(\mu) = 2m_\pi^2/\overline{m}_b(\mu)(\overline{m}_u(\mu) + \overline{m}_d(\mu))$. The other quantities are defined in Ref.[7]

Terms proportional to b_i in eq.(6) are referred as annihilation contributions. These terms have end-point divergences of the form $X_A = \int_0^1 \phi_\sigma(y)dy/(1-y)$ and need regularization. The term proportional to f^{II} also has a similar end-point divergence X_H. These divergences associated with $1/m_{b,c}$ corrections indicate in a way the incompleteness of factorization calculation. In Ref.[7] these end-point divergences are parameterized as $\ln(m_b/\Lambda_h)(1 + \rho_{A,H}e^{i\phi_{A,H}})$. An idea of the size of the corrections can be obtained by varying $\rho_{A,H}$ and $\phi_{A,H}$.

To finally obtain numerical results, one needs to know the CKM and the hadronic parameters in the above amplitudes. There are considerable progresses in the determination of the

CKM parameters[5, 11]. We will use the central values given in Ref.[5] with $s_{12} = 0.2243$, $s_{23} = 0.0413$, $s_{13} = 0.0037$ and the CP violating phase $\gamma(\delta_{13}) = 60^\circ$ for illustration. For the hadronic parameters we use the "default" values given in Ref.[7]. The results for the ratio R_B are listed in Table I. In the table, we also list the values obtained by using naive factorization[12] for comparison. We see that the α_s order correction is at the order of 10% and can be as large as 30% if the involved hadronic parameters vary within a reasonable range. One also notes that the term f_I generates an absorptive part which is absent in naive factorization calculations.

One can see from Table I that the theoretical calculation is in qualitative agreement with the data that there is no $\Delta I = 1/2$ rule in $B \to \pi\pi$ decays. This is expected since that in factorization calculations the isospin does not play a special role. The leading operators, i.e. the tree operators, which contribute to $B \to \pi\pi$, contain both $\Delta I = 1/2$ and 3/2 pieces with similar weights, the amplitudes for isospin I =0 and I =2 are therefore expected to have similar sizes.

We now consider the situation for $D \to \pi\pi$ decays. In Ref. [15] Sannino noticed the violation of the $\Delta I = 1/2$ rule in $D \to \pi\pi$ and tried to understand it based on the effective weak Hamiltonian. Assuming that the hadronic matrix elements M_2, M_0 and $\tilde{M}_0$ defined in Ref.[15] are all equal, numerically $R_D \sim 0.32 \sim 0.44$ which is not too far away from data.

The c-quark is not as heavy as the b-quark, factorization calculation for $D \to \pi\pi$ may not work as well as for $B \to \pi\pi$. We however expect that a theoretical calculation based on QCD factorization scheme can still provide some crude estimate. We therefore also use the QCD factorization for $D \to \pi\pi$. In this calculation we will use the Wilson coefficients given in Ref.[16]. We have also checked the dependence on the renormalization scale and find that the result on R_D does not change much with respect to the scales. The numerical results are listed in Table I. We see that the situation is similar to that for $B \to \pi\pi$, there is no $\Delta I = 1/2$ rule.

The calculation for $K \to \pi\pi$ using factorization becomes very questionable. Nevertheless, attempts have been made to evaluate the amplitudes. Buras et al.[17] evaluated the ratio R_K based on the large N expansion approach with

$$R_K = \left| \frac{A_0}{A_2} \right| = \frac{c_1(\mu) < O_1(\mu) >_0 + c_2(\mu) < O_2(\mu) >_0 + \sum_{i=3}^{6} c_i(\mu) < O_i(\mu) >_0}{c_1(\mu) < O_1(\mu) >_2 + c_2(\mu) < O_2(\mu) >_2} \tag{7}$$

and the hadronic matrix elements can be parameterized as

$$
\begin{aligned}
&< O_1 >_0= -\frac{1}{9}XB_1^{(1/2)}, \;\; < O_2 >_0= \frac{5}{9}XB_2^{(1/2)},\\
&< O_3 >_0= \frac{1}{3}XB_3^{(1/2)}, \;\; < O_4 >_0=< O_3 >_0 + < O_2 >_0 - < O_1 >_0,\\
&< O_5 >_0= \frac{1}{3} < O_6 >_0, \;\; < O_1 >_2=< O_2 >_2= \frac{4\sqrt{2}}{9}XB_1^{(3/2)},\\
&< O_6 >_0= -4\sqrt{\frac{3}{2}}[\frac{m_K^2}{\overline{m}_s(\mu)+\overline{m}_d(\mu)}]^2\frac{F_\pi}{\kappa}B_6^{(1/2)},
\end{aligned}
\tag{8}
$$

where $X = \sqrt{\frac{3}{2}}F_\pi(m_K^2 - m_\pi^2)$, $\kappa = F_\pi/(F_K - F_\pi)$ and B_i are the hadronic parameters to be determined either by fitting experimental data or invoking phenomenological models. By fitting data they obtained a set of the hadronic parameters as $B_1^{(3/2)} = 0.48$, $B_1^{(1/2)} = 10$, $B_2^{(1/2)} = 5$ and $B_3^{(1/2)} = B_6^{(1/2)} = 1$. The value of R_K calculated in this approach agrees with data. One notes that several bag parameters B_i are substantially away from the naive value of $B_i = 1$. Thus one expects that some non-perturbative effects and the final state interaction effects are involved altogether in the parameters.

We note that although theoretical calculations agree with data, one can be confirmed with the fact that there is not a $\Delta I = 1/2$ rule in $B(D) \to \pi\pi$ decays, there is a large difference in $\bar{B}^0(D^0) \to \pi^0\pi^0$ branching ratio between naive theoretical estimates and data. It is important to see if these discrepancies can be explained.

A possible source may be due to uncertainties in the factorization calculation, in particular corrections of $1/m_{b,c}$. As pointed out earlier that the end-point divergences appearing in the hard scattering and annihilation signal incompleteness of the $1/m_{b,c}$ corrections. A complete treatment of these effects are beyond the scope of this work. To have some idea about the effects of $1/m_{b,c}$ we have carried out a calculation by varying the parameters ρ_{AH} from 0 to 3 and $\phi_{A,H}$ from 0 to 2π. We find that the changes on the branching ratios of $D^+(B^+) \to \pi^+\pi^0$ and $D^0(B^0) \to \pi^+\pi^-$ are in the range of 20% to 30%, and the changes on $D^0(B^0) \to \pi^0\pi^0$ can be dramatic (a factor of 2 to 3). However, it is still not possible to bring $D^0(B^0) \to \pi^0\pi^0$ branching ratios to their experimental values. One may need to consider other effects which are not included in the factorization approximation.

To this end we consider long distant final state interaction effects. If the differences are really due to FSI effects, it should not only explain the differences mentioned above, but should also explain why there is a $\Delta I = 1/2$ rule in $K \to \pi\pi$, but not in $B(D) \to \pi\pi$.

To explain the $\Delta I = 1/2$ rule in K-decays, Isgur et al. [2] proposed a possible mechanism for the smallness of R_K ($\sim 1/22$) that before the two pions produced from the weak decay of the kaon fly apart to become free particles, they reside in bound states of either $I = 0$ or $I = 2$. Due to different interactions for the two different isospin states, the wavefunctions at origin for $I = 0$ and $I = 2$ states are distorted differently. The hadronic matrix elements of the two isospin states would undergo an enhancement or a suppression as[2]

$$\frac{< (\pi\pi)_{I=0} \mid H_W^{1/2} \mid K >}{< (\pi\pi)_{I=2} \mid H_W^{3/2} \mid K >} = (\frac{d_0}{d_2})^{\frac{1}{2}} \frac{< (\pi\pi)_{I=0}^{free} \mid H_W^{1/2} \mid K >}{< (\pi\pi)_{I=2}^{free} \mid H_W^{3/2} \mid K >}. \tag{9}$$

Here the distorting parameter d_I is defined as $d_I = |\psi^{true}(0)/\psi^{free}(0)|^2$.

The general requirement for the interaction potential is that the $I = 0$ channel experiences an attractive interaction whereas the $I = 2$ channel experiences a repulsive one. We note that a simple square well potential of the form $U\mathbf{I_1} \cdot \mathbf{I_2} = (1/2)U[I(I+1) - I_1(I_1+1) - I_2(I_2+1)]$ with $U > 0$ can result in the required potential form.

Fitting data on the low-energy $\pi - \pi$ scattering phase shifts, the parameters of V_I and a_I can be determined. The distortion due the potential can result in an enhancement of the channel of the $\Delta I = 1/2$ over the $\Delta I = 3/2$ by a factor $r = (d_0/d_2)^{1/2}$ for $K \to \pi\pi$ which can be as large as $9 \sim 10$. For the isospin correlated potential $U\mathbf{I_1} \cdot \mathbf{I_2}$ with $U = 0.4$ GeV and potential range $a_0 = a_2 = 0.8$ fm, the enhancement factor r is 9. This factor can make up the gap between theory and experimental data leading to the $\Delta I = 1/2$ rule in the case of $K \to \pi\pi$. One can also try more complicated potential forms, such as Gaussian-type potentials as studied in Ref.[2], but the qualitative features are unchanged.

Since the final two pions produced from B and D decays have similar isospin structure except that their kinetic energies are much higher, they should experience similar effects due to the potential. However, since the two pions have much higher kinetic energies they have much shorter time to interact with each other before emerging out of the bound states to produce large effects. More specifically, in the D and B cases, the momenta $|\mathbf{k}| = \frac{1}{2}\sqrt{(M_{D(B)}^2 - M_\pi^2)}$ are much larger than the potential energies $|V_I|$, one can easily verify that $d_0 \approx d_2 \approx 1$. Even though for the B and D decays, the two pions fly very fast and the non-relativistic quantum mechanical scenario may not produce accurate numbers, the qualitative conclusion about smallness of the bound state effects, however, should hold. We therefore see that this mechanism can easily explain why there is a $\Delta I = 1/2$ rule for $K \to \pi\pi$ but not for $B(D) \to \pi\pi$.

We now discuss the FSI phases. Using experimental data, we obtain $\cos\delta_D$ and $\cos\delta_D$ which are shown in eq.(5). These results indicate that the difference of the phase shifts δ_P is sizeable. Although factorization amplitudes in eq.(6) have included some re-scattering effects at quark level, they are not large enough to explain data. Factorization calculations using the default values for relevant hadronic parameters would give a $\cos\delta_P$ to be very close to 1. There are uncertainties in factorization calculations in regularizing the end-point divergences. As mentioned earlier, we have checked numerically that within reasonably allowed parameter space, these contributions cannot generate large enough phase shifts to reproduce data.

The distorting effects in eq.(9) discussed earlier can also generate a phase shift. At the kaon energy, it is possible to generate the required phase shift for $K \to \pi\pi$. However these phase shifts go to zero as the energies become much larger than the potentials. For B and D decays, the phase shifts are practically zero because the pions from $D(B)$ decays have high energy and pass through the potential in too short a time to produce any significant phase shifts. There is a need of additional effects. The effects producing large phase shifts in D and B decays must be different from the mechanism in Ref.[2] which is indeed available, so that at higher energies more channels become active in producing absorptive part of decay amplitudes and therefore phase shifts[13, 14].

From the calculated branching ratios of $D^0 \to \pi\pi$ and $B^0 \to \pi\pi$ in Table I, we note that the value for $\pi^+\pi^-$ is obviously higher than data, whereas the value for $\pi^0\pi^0$ is significantly lower than data. However the sum of $BR(P^0 \to \pi^+\pi^-)$ and $BR(P^0 \to \pi^0\pi^0)$, and also $BR(P^- \to \pi^-\pi^0)$ are close to experimental values in both D and B cases. Since only these sums determine the ratios R_P (see eq.(2)), the theoretical calculations are close to data. One should keep in mind that there are uncertainties in many of the hadronic parameters. The theoretical predictions for the branching ratios can change, but it is difficult to generate large enough branching ratios for $B^0(D^0) \to \pi^0\pi^0$. If re-scattering of the two pions can change the higher valued $\pi^+\pi^-$ into the lower valued $\pi^0\pi^0$ after the pions come out of the bound states as discussed above, it may be able to produce the correct values to meet data. In fact large phase shifts due to re-scattering in $B(D)$ decay into two light mesons have been noticed before[13, 14]. We have carried out a simple exercise similar to some work in Ref.[13] taking the magnitudes of $A_{0,2}$ as determined by factorization calculations and δ as a free parameter to fit data. We find that with δ_D equal to 0.45π, $\cos\delta_D$ is close

to the central value of the data. In the case of $B \to \pi\pi$, with the default values for the hadronic parameters, $\delta_B = 0.30\pi$ can reproduce a $\cos\delta_B$ which makes the branching ratio of $B \to \pi^0\pi^0$ to be close to the experimental central value. In this case both amplitudes $A_{0,2}$ are slightly higher than data, but can easily be made to agree with data if a slightly smaller form factor $F^{B\to\pi} = 0.23$ is used instead of the default value of 0.28. A more precise theoretical evaluation of the re-scattering phases would be difficult because the mechanisms are not fully understood yet.

We finally make some comments about isospin analysis and CP violation. The FSI phases are very important for the study of CP violations. Sizeable CP asymmetry can occur in $B \to \pi\pi$ decays. We therefore will concentrate on $B \to \pi\pi$ decays. The isospin amplitude given before can be further decomposed into different components according to the CKM matrix elements associated with

$$A_i = V_{ub}V_{ud}^* a_i^T + V_{tb}V_{td}^* a_i^P. \tag{10}$$

The components a_i^T mainly come from tree amplitudes by exchange W-boson but also small corrections from loop-induced c and u penguin contributions, and a_i^P are induced at loop level by c and t penguins.

In general there can be 4 complex hadronic parameters $a_{1,2}^T$ and $a_{0,2}^P$ to describe $B \to \pi\pi$ decays. Among them one can always set one of the components to be real, there are actually only 7 independent parameters. Furthermore in the SM, the leading order from c and t penguin contributions to a_2^P are dominated by t penguin from operator $O_{9,10}$ which have the same Lorentz structure as the tree operators. One has[18] $a_2^P = 3/2(c_9 + c_{10})/(c_1 + c_2)a_2^T$. Neglecting other smaller contributions to $a_2^{T,P}$ components, one only needs five hadronic parameters to describe $B \to \pi\pi$ decays. This is an important fact which has interesting implications.

One of them is that it can be used to determine CP violating phase γ if the 7 possible experimental observables in $B \to \pi\pi$ decays are all measured (the 3 branching ratios, the 2 direct CP violation ($B^- \to \pi^-\pi^0$ has very small CP violation in the SM and probably cannot be measured), and 2 mixing induced CP violation for $\bar{B}^0 \to \pi^+\pi^-, \pi^-\pi^0$), the 7 observables can determine the five hadronic parameters and two CKM parameters ρ and η. We need to wait for more data to carry out a full analysis. If one takes the CKM parameters

as determined by other data, the hadronic parameters can already be extracted.

In obtaining the value $\cos\delta_B$ in eq.(5), we have neglected possible CP violating effect in the isospin amplitudes. We therefore cannot determine a_0^T and a_0^P separately. One needs more data points to obtain more detailed information about the size of the decay amplitudes and also phases. Fortunately experimentally, besides the branching ratios listed in Table I for $B \to \pi\pi$, there are also some measurements on the direct CP violating parameter A_{CP} and mixing induced CP violating parameter S_{CP}[6],

$$\begin{aligned}
&A_{CP}(\bar{B}^0 \to \pi^+\pi^-) = 0.37 \pm 0.11, \quad S_{CP}(\bar{B}^0 \to \pi^+\pi^-) = -0.61 \pm 0.14 \\
&A_{CP}(\bar{B}^0 \to \pi^0\pi^0) = 0.28 \pm 0.39.
\end{aligned} \tag{11}$$

Using five data points, the three branching ratios and the CP asymmetry parameters A_{CP} and S_{CP} for $\bar{B}^0 \to \pi^+\pi^-$, as input we can solve for the hadronic amplitudes. We have

$$\begin{aligned}
&1) \qquad a_2^T = 0.366A, \;\; a_0^T = 0.276e^{-i48.5^\circ}A, \;\; a_0^P = 0.067e^{-i71.1^\circ}A; \\
&2) \qquad a_2^T = 0.366A, \;\; a_0^T = 0.289e^{i54.3^\circ}A, \;\; a_0^P = 0.065e^{-i16.6^\circ}A.
\end{aligned} \tag{12}$$

Here $A = \sqrt{BR(B^- \to \pi^-\pi^0)\Gamma_{B_{total}}}$ Factorization calculations give different values, in particular the phases. Improved theoretical framework of calculating the amplitudes is needed.

If the penguin amplitude A_0^P is neglected, there is no direct CP violation, one can use eq. (4) to determine the FSI phase. Since A_0^P is small compared with the three amplitudes, the use of eq. (4) is a good approximation. We note that the two solutions have different FSI phases for each individual amplitude, but the phase difference of A_0^T and A_2^T are similar in size for solutions 1) and 2) and different in signs, Since the cosine function is not sensitive to the sign, one would obtain similar $\cos\delta_B$. One needs to find some observables which can distinguish these solutions. We indeed find that CP violation in A_{CP} of $\bar{B}^0 \to \pi^0\pi^0$ depends on the sign. We have not used the data as input because the error is large there. Using the above amplitudes and phase we can predict its value. We obtain $A_{CP}(\pi^0\pi^0) = -0.60$ and 0.18 for solutions 1) and 2), respectively. The solution 2) obtains a value close to the current central value. But due to the large error at present, it is too early to decide which solution is the correct one. When more precise data become available one can distinguish these solutions and obtain more detailed information about the isospin amplitudes.

In summary in this work we have analyzed the isospin amplitudes due to $\Delta I = 1/2$ and $\Delta I = 3/2$ interactions in the $K \to \pi\pi$, $D \to \pi\pi$ and $B \to \pi\pi$ decays. Experimental data clearly show that unlike the situation for $K \to \pi\pi$, there is no $\Delta I = 1/2$ rule in $B(D) \to \pi\pi$ decays. Theoretical calculations using the naive factorization and the QCD factorization are consistent with data about the violation of $\Delta I = 1/2$ rule, but there are difficulties to obtain correct phases in the amplitudes and branching ratio for $BR(\bar{B}^0 \to \pi^0\pi^0)$.

The question why there is a $\Delta I = 1/2$ rule in $K \to \pi\pi$ whereas not for $B(D) \to \pi\pi$ can be interpreted by the re-scattering of the final two pions due to a isospin correlated interaction proposed by Isgur et al. We note that the potential of the form $U\mathbf{I_1} \cdot \mathbf{I_2} = (1/2)U[I(I+1) - I_1(I_1+1) - I_2(I_2+1)]$ can play such a role. To fully explain $B(D) \to \pi\pi$ data, there is also the need of re-scattering after the pions come out of the bound states. These additional phases are not precisely calculable at present.

We have also shown that isospin analysis for $B \to \pi\pi$ provides valuable information about CP violation in B decays and hadronic amplitudes.

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[1] M. Gaillard and B. Lee, Phys. Rev. Lett. **33**, (1974)108; G. Altarelli and L. Maiani, Phys. Lett. **B52**, (1974)351.

[2] N. Isgur, K. Maltman, J. Weinstein and T. Barnes, Phys. Rev. Lett **64**, (1990)161.

[3] M. Shifman, A. Vainstein and V. Zakharov, Nucl.Phys.**B120**, (1977)316.

[4] B. Stech, Phys. Rev. **D36**(1987)975.

[5] Particle Data Group, Phys. Lett. **B592**, 1(2004).

[6] B. Aubert et al.[Barbar Collab.], hep-ex/0308012; K. Abe et al.[Belle Collab.], hep-ex/0308040. See Heavy Flavor Averaging Group [HFAG], http://www.slac.stanford.edu/xorg/hfag for a recent average.

[7] M. Beneke, G. Buchalla, M. Neubert and C. Sachrajda, Nucl. Phys. **B606**, (2001)245; Phys. Rev. Lett. **83**, 1914(1999); M. Beneke and M. Neubert, Nucl. Phys. **B675**, 333(2003).

[8] D. Du, D. Yang and G. Zhu, Phys. Lett. **B488**, (2000)46; D. Du, D. Yang and G. Zhu, Phys.

Rev. **D64**, (2001)014036; D. Du, H. Gong, J. Sun, D. Yang and G. Zhu, Phys. Rev. **D65**, (2002)074001.

[9] Y.-Y. Keum, H.-N. Li and A. Sanda, Phys. Lett. **B504**, 6(2001); Phys. Rev. **D63**, 054008(2001).

[10] C. Bauer, D. Pirjol, I. Rothstein, I. Stewart, Phys. Rev.**D70**, 054015(2004).

[11] M. Battaglia et al., hep-ph/0304132.

[12] M. Bauer, B. Stech and M. Wirbel, Z. Phys. **C29**, (1985)637; **C34**, (1987)103; A. Ali, G. Kramer and C. Lu, Phys. Rev. **D59**, 014005(1999); **D58**, (1998)094009.

[13] K. Pham, Phys. Lett. **193**,(1987 331; H. Lipkin, Phys. Rev. Lett. **44**, (1980)710; Y. Dai et al. Phys. Rev. **D60**, (1999)014014; X. Li and B. Zou, Phys. Rev. **D57**, (1998)1518; N.G. Deshpande et al., Phys. Rev. Lett. **82**, (1999)2240; Z.-H. Li, F. Yuan and H.-Q. Zheng, Commun. Theor. Phys. **34**, (2000)567; W.-S. Hou and K.-C. Yang, Phys. Rev. Lett. **84**, (2000)4806; C.-K. Chua, W.-S. Hou and K.-C. Yang, Mod. Phys. Lett. **A18**, (2003)1763; X.-G. He et al., Phys. Rev. **D64**, (2001)034002; H.-K. Fu et al., hep-ph/0206199, Chin. J. Phys. **41**, (2003)601 ; hep-ph/0304242 (Phys. Rev. D in press); M. Gronau aned J. Rosner, Phys. Rev. **D58**, (1998)113005; **D65**, (2002)093012; C.-W. Chiang, M. Gronau and J. Rosner, Phys. Rev. **D68**, (2003)074012.

[14] W. Chiang et al., Phys. Rev. **D70**, 034020(2004); A.J. Buras et al., Phys. Rev. Lett. **92**, 101804(2004); Y.L. Wu and Y.F. Zhou, Phys. Rev. **D71**, 021701(2005); Y.-Y. Charng and H.-N. Li, Phys. Rev. **D71**, 014036(2005); Xiao-Gang He and B. McKellar, hep-ph/0410098; A. Y. Parkhomenko, hep-ph/0411061; H.Y. Cheng, C. K. Chua and A. Soni, Phys. Rev. **D71**, 014030(2005).

[15] F. Sannino, Phys. Lett. **B322**, (1994)259.

[16] F. Buccella, M. Lusignoli, G. Mangano, G. Miele, A. Pugliese and P. Santorelli, Phys. Lett. **B302**, (1993)312.

[17] G. Buchalla, A. Buras and M. Lautenbacher, Rev. Mod. Phys. **D68**, (1996)1125.

[18] M. Neubert and J. Rosner, Phys. Lett. **B441**, 403(1998).